Principles of Program Analysis

An overview of approaches

The Nature of static analysis --- approximation

- Static program analysis --- predict the dynamic behavior of programs without running them
  - Example: at each program execution step, what is the value of each variable?

```
int x, y, z;
read(&x);
if (x>0) { y=x; z = 1}
else { y= - x; z = 2}
```

  - The question cannot be answered precisely b/c the program input is unknown
    - We don't know the value of x, and therefore cannot predict which branch will be taken (whether the value of x is greater than 0)
    - However, we can predict all the possible values for z and that y is >= 0 at the end of code.
  - Program analysis approach: tries to give approximate answers; tries to prove properties of program entities (variables, functions, types)

The Nature of Approximation --- may and must analysis

- Since the behavior of programs cannot be predicted precisely, there are two ways to approximate
  - Over approximation: what *may* happen when all possible inputs are considered?
    - The answer is a superset of what happens at runtime
  - Under approximation: what *must* always happen in spite of different inputs?
    - The answer is a subset of what happens at runtime
- What approximation to use depends on what the results will be used for
  - Should always err on the safe side
    - Example: if we want to remove all useless evaluations in the program, should we find evaluations that may or must be useless?
- The relation between may and must analysis
  - Find all evaluations that are always useless (must analysis)

  <=> find all evaluations that may be useful (may analysis)

The Precision of Approximation --- How input sensitive is the analysis?

- Flow sensitivity: Is solution sensitive to the control flow within a function?
  - Flow-insensitive analysis
    - Example: what variables may be accessed by a code?
    - Solution: find all the variables that appear in the code
  - Flow sensitive analysis
    - Example: what values a variable may have at each program point
    - A different solution must be found for each program point
- Context sensitivity: Is solution sensitive to the calling context of a function?
  - Context-insensitive: a single solution is computed for each function, no matter who calls the function
  - Context-sensitive: different solutions are computed for different chains of callers
- Path sensitivity? Is solution sensitive to different execution paths of a program?
  - Path sensitive: different solutions are computed for different paths from program entry to each statement

Scopes of Program Analysis

- What code are examined to find the solution?
  - Local analysis
    - Operate on a straight-line sequence of statements (a basic block)
    - Often used as basis for more advanced analysis approaches
  - Regional analysis
    - Operate on code with limited control flow, e.g., loops, conditionals
    - Useful for special-purpose optimizations (e.g., loop optimizations)
  - Global (intra-procedural) analysis
    - Operate on a single procedure/subroutine/function
    - Required by most flow-sensitive analysis problems
  - Whole-program (inter-procedural) analysis
    - Operate on an entire program (all sources must be available)
    - Required by context and path sensitive analysis

Common Approaches to Program Analysis

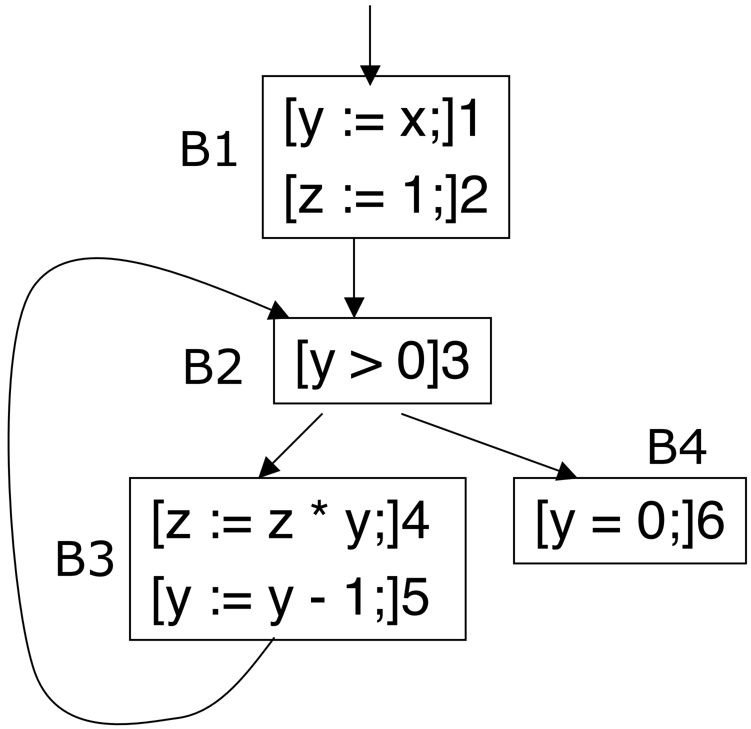
- A family of techniques
  - Data flow analysis: operate on control-flow graph
    - Define a set of data to evaluate at entry and exit of each basic block
    - evaluate the flow of data between pred/succ basic blocks
  - Constraint based analysis
    - For each program entity to be analyzed, define a set of constraints involving information of interest
    - Solve the constraint system via mathematical approaches
  - Abstract interpretation
    - Define a set of data to evaluate at each program point; Map each statement/construct to a finite sequence of semantic actions
    - Statically interpret each instruction in program execution order
  - Type and effect systems
    - Categorize different properties into a collection of types/groups
    - Infer the type/group of each program entity from how it is used
- Techniques differ in algorithmic methods, semantic foundations, language paradigms

Example Dataflow Analysis
Reaching Definitions

Example program with labels

```
[y := x;]1
[z := 1;]2
while [y > 0]3  {
  [z := z * y;]4
  [y := y - 1;]5
}
[y = 0;]6
```

Control-flow graph



An assignment [x := a]i reaches j if there is an execution path from entry to j where x was last assigned at i

Reaching Definition Analysis
The best solution

← {(x, ?), (y, ?), (z, ?)}

[y := x;]1

← {(x, ?), (y, 1), (z, ?)}

[z := 1;]2

← {(x, ?), (y, 1), (y, 5), (z, 2), (z, 4)}

while [y > 0]3 {

← {(x, ?), (y, 1), (y, 5), (z, 2), (z, 4)}

[z := z * y;]4

← {(x, ?), (y, 1), (y, 5), (z, 4)}

[y := y - 1;]5

← {(x, ?), (y, 5), (z, 4)}

}

← {(x, ?), (y, 1), (y, 5), (z, 2), (z, 4)}

[y = 0;]6

← {(x, ?), (y, 6), (z, 2), (z, 4)}

Solving the data-flow problem Reaching definitions

- Domain of analysis
  - The set of all definition points in a procedure/function
  - A definition point d of variable v reaches CFG point p iff there is a path from d to p along which v is not redefined
  - At any CFG point p, what definition points can reach p?
- Reaching definition analysis can be used in
  - Building dafa-flow graphs
    - Provide info where each operand is defined
  - SSA (static single assignment) construction
    - A representation that encodes both control and data flow of a procdure
- For each basic block n, let
  - DEDef(n)= definition points whose variables are not redefined in n
  - DefKill(n)= definitions obscured by redefinition of the same name in n d

Goal: evaluate all definition points that can reach entry of n

- $RD(n) = \bigcup_{m \in pred(n)} (DEDef(m) \cup (RD(m) - DefKill(m)))$

Dataflow Analysis Algorithm Computing Reaching Definitions

- For each basic block n, compute
  - DEDef(n)= definition points whose variables are not redefined in n
  - DefKill(n)= definitions obscured by redefinition of the same name in n d

Goal: evaluate all definition points that can reach entry of n

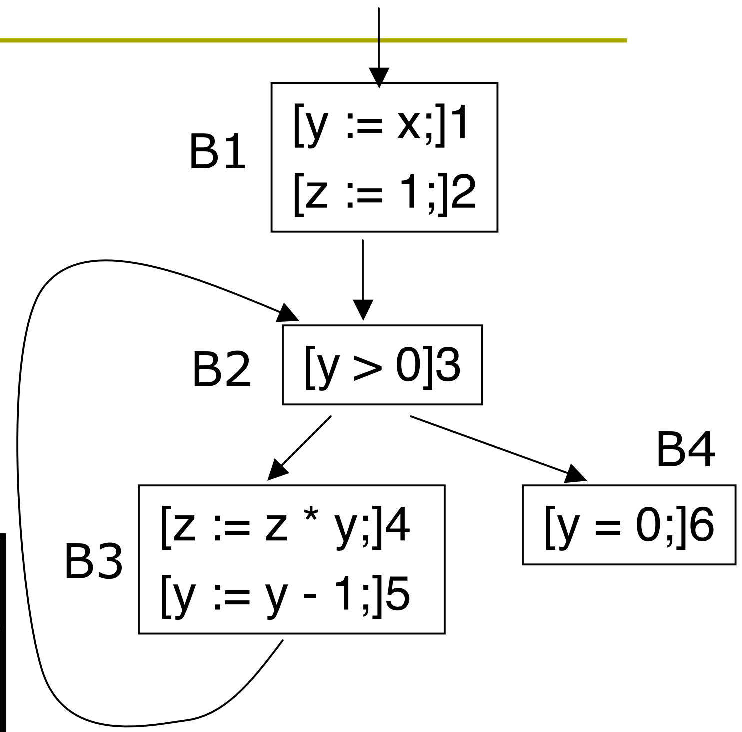
$$RD(n) = \bigcup_{m \in pred(n)} (DEDef(m) \cup (RD(m) - DefKill(m)))$$

```
for each basic block bi
    compute DEDef(bi) and DefKill(bi)
    RD(bi) := ∅
for (changed := true; changed; )
    changed = false
    for each basic block bi
        old = RD(bi)
        RD(bi)= ∪ (DEDef(m) ∪ (RD(m) - DefKill(m))
             m∈pred(bi)
        if (RD(bi) != old) changed := true
```

Example solution: reaching definition analysis

```
[y := x;]1
[z := 1;]2
while [y > 0]3  {
   [z := z * y;]4
   [y := y - 1;]5
 }
 [y = 0;]6
```

	DEDef	DefKill	RD	RD	RD
B1	1,2	5,6,4	∅	∅	∅
B2	∅	∅	∅	1,2,4,5	1,2,4,5
B3	4,5	1,2,6	∅	1,2,4,5	1,2,4,5
B4	6	1	∅	1,2,4,5	1,2,4,5



Domain: 1 2 4 5 6
y z z y y

Example Constraint-based Analysis Loop dependence analysis

Example code

```
for (i=0;i<N;++i) {
    for (j=0;j<N;++j) {
          C[i*N+j]=(C[(i-1)*N+j]+C[i*N+j-1])/2;
   }
}
```

A loop iteration (i,j) depends on another iteration (i’,j’) if it uses the value computed by (i’,j’) or if it writes to a common location written by (i’,j’)

If a loop iteration (i,j) depends on iteration (i’,j’), the ordering of the two iterations cannot be switched.

Loop dependence analysis
Solving a system of equations

Example code

```
for (i=0;i<N;++i) {
    for (j=0;j<N;++j) {
          C[i*N+j]=(C[(i-1)*N+j]+C[i*N+j-1])/2;
  }
}
```

- A loop iteration (i,j) depends on another iteration (i’,j’) if (i’,j’) computes the value used by (i,j), that is
  - If (C[i’*N+j’], C[(i-1)*N+j])

    (C[i’*N+j’], C[i*N+j-1])

    or (C[i’*N+j’], C[I*N+j]) refer to the same location.
  - That is, if i’*N+j’ = (i-1)*N+j, i’*N+j’ = i*N+j-1,

    or i’*N+j’=i*N+j

Example abstract interpretation analysis Points-to analysis

Example program with labels

```
struct Cell {
  int val;
  struct Cell* next;
} *h, *t, *p;
 [h = t = NULL;]1
 for (int [i=0]2; [i<N]3; [++i]4) {
    [p = new Cell(i,NULL);]5
    if ([h == NULL]6)
      [h = t = p;]7
    else {
      [t->next = p; t = p;]8
    }
 }
```

- Define the data to evaluate
  - A set of locations for each pointer variable
  - Keep track of constant values for non-pointer variables
- Define a semantic action for each statement
  - Modifies the location set of pointer variables
  - Allocate new locations
    - Limit the number of locations for each stmt
  - Control flow (conditionals, loops, and function calls)
    - Assume all branches are taken when not sure

What locations can each pointer variable points to? (can they point to the same location?)

Abstract interpretation of points-to locations

```
[h = t = NULL;]1
[i=0;]2
if [i<N]3;
[p = new Cell(i,NULL);]5
if ([h == NULL]6)
      [h = t = p;]7
[++i]4
if [i<N]3;
[p = new Cell(i,NULL);]5
if ([h == NULL]6)
   else {[t->next = p; t = p;]8
[++i]4   if [i<N]3;
```

```
h -> 0  t -> 0  p -> ?
h -> 0  t -> 0  p -> ?
h -> 0  t -> 0  p -> ?
h -> 0  t -> 0  p -> new[5]
h -> 0  t -> 0  p -> new[5]
h ->new[5]  t ->new[5]  p -> new[5]
h ->new[5]  t ->new[5]  p -> new[5]
h ->{0,new[5]}  t ->{0,new[5]}  p -> {?,new[5]}
h ->{0,new[5]}  t ->{0,new[5]}  p -> new[5]
h ->{0,new[5]}  t ->{0,new[5]}  p -> new[5]
h ->{0,new[5]}  t ->new[5]  p -> new[5]
Exit loop if evaluation has stopped changing
h ->{0,new[5]}  t ->{0,new[5]}  p -> {?,new[5]}
```

Abstract Interpretation

```
AbstractInterpretation(op)
   if (is_assignment(op))
         modify_memory_from_assignment(memory(op), op)
   else if (is_conditional(op)) then
        AbstractInterpretation(cond(op));
        AbstractInterpretation(tree_branch(op));
        AbstractInterpretation(false_branch(op));
   else if (is_loop(op)) then
         repeat
            start_monitor_all_changes(memory(stmts(op)))
            AbstractInterpretation(stmts(op))
         until  nothing changes in memory(stmts(op))
   else if (is_procedural_call(op)) then
         setup_parameters_and_return(op);
         AbstractInterpretation(body(op));
   else …
```

Example Solution Abstract Interpretation

```
struct Cell {
   int val;
   struct Cell* next;
} *h, *t, *p;
 [h = t = NULL;]1
 for (int [i=0]2; [i<N]3; [++i]4) {
     [p = new Cell(i,NULL);]5
     if ([h == NULL]6)
        [h = t = p;]7
     else {
        [t->next = p; t = p;]8
     }
 }
```

Domain: h,t,p

0	h->? t->? p->?	
1	h->0 t->0 p->?	
2	h->0 t->0 p->?	
3	h->0 t->0 p->?	h->{0,new[5]} t->{0,new[5]} p->{?,new[5]}
5	h->0 t->0 p->new[5]	h->{0,new[5]} t->{0,new[5]} p->new[5]
6	h->0 t->0 p->new[5]	h->{0,new[5]} t->{0,new[5]} p->new[5]
7	h->new[5] t->new[5] p->new[5]	h->new[5] t->new[5] p->new[5]
8		h->new[5] t->new[5] p->new[5]
4	h->new[5] t->new[5] p->new[5]	h->new[5] t->new[5] p->new[5]

Example type and effect analysis Points-to analysis

Example program with labels

```
struct Cell {
  int val;
  struct Cell* next;
} *h, *t, *p;
 [h = t = NULL;]1
 for (int [i=0]2; [i<N]3; [++i]4) {
    [p = new Cell(i,NULL);]5
    if ([h == NULL]6)
      [h = t = p;]7
    else {
      [t->next = p; t = p;]8
    }
 }
```

- The type domain: locations
  - Each statement that allocates a new location
  - Each variable that has a location
- Examine each statement and infer a type (a group of locations) for each pointer variable
  - Each pointer variable can have only a single type, no matter where it appears
    - Flow insensitive
- If a distinct type is inferred for each expression, then analysis is flow sensitive

What locations can each pointer variable points to? (can they point to the same location?)

Applying type and effect approach to points-to analysis

Example program with labels

```
struct Cell {
   int val;
   struct Cell* next;
} *h, *t, *p;
 [h = t = NULL;]1
 for (int [i=0]2; [i<N]3; [++i]4) {
     [p = new Cell(i,NULL);]5
     if ([h == NULL]6)
        [h = t = p;]7
     else {
        [t->next = p; t = p;]8
     }
 }
```

- The type domain includes
  - NULL, new[5]
- Examine the program text and union all types (locations) for each variable
  - [h=t=NULL]1
    - H->NULL; t->NULL;
  - [p = new Cell(i,NULL);]5
    - P-> new[5]
  - [h = t = p;]7 and [t = p;]8
    - Type(p) is a subset of Type(h)
    - Type(p) is a subset of Type(t)
- Result:
  - h=> {NULL,new[5]}
  - t=> {NULL, new[5]}
  - p=> new[5]
- Key: define typing rules

Type Inference based points-to analysis

Flow-insensitive type inference:

For each pointer variable v do

Type(v) = {}

For each operation that assigns a new set of locations L to pointer v

do

Type(v) = Type(v) ∪ L

0	h->{} t->{} p->{}
1	h->{0} t->{0} p->{}
2	h->{0} t->{0} p->{}
3	h->{0} t->{0} p->{}
4	h->{0} t->{0} p->{new[5]}
5	h->{0} t->{0} p->{new[5]}
6	h->{0} t->{0} p->{new[5]}
7	h->{0,new[5]} t->{0, new[5]} p->{new[5]}
8	h->{0,new[5]} t->{0, new[5]} p->{new[5]}