

**Rotation Points From  
Motion Capture Data Using a Closed Form Solution**

by

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Thesis directed by Professor Sudhanshu Kumar Semwal

Four new closed-form methods are present to find rotation points of a skeleton from motion capture data. A generic skeleton can be directly extracted from noisy data with no previous knowledge of skeleton measurements. The new methods are ten times faster than the next fastest and a hundred times faster than the most widely accepted method. Two phases are used to produce an accurate skeleton of the captured data. The *first phase*, fitting the skeleton, is robust even with noisy motion capture data. The formulae use an asymptotically unbiased version of the Generalized Delogne-Kása (GDKE) Hyperspherical Estimation (first estimator: UGDK). The second estimator takes advantage of multiple markers located at different distances from the rotation point (MGDK) thereby increasing accuracy. The third estimator removes singularities to allow for cylindrical joint motion (SGDK). The fourth estimator incrementally improves an answer and has advantages of constant memory requirements suitable for firmware applications (IGDK). The UGDK produces the answer faster than any previous algorithm and with the same efficiency with respect to the Cramér-Rao Lower Bound for fitting spheres and circles. The UGDK method significantly reduces the amount of work needed for calculating rotation points by only requiring  $26N$  flops for each joint. The next fastest method, Linear Least-Squares requires  $236N$  flops. In-depth statistical analysis shows the UGDK method converges to the actual rotation point with an error of  $O(\sigma/\sqrt{N})$  improving on the GDKE's biased answer of  $O(\sigma)$ . The *second phase* is a real-time algorithm to draw the

skeleton at each time frame with as little as one point on a segment. This speedy method, on the order of the number of segments, aids the realism of motion data animation by allowing for the subtle nuances of each time frame to be displayed. Flexibility of motion is displayed in detail as the figure follows the captured motion more closely. With the reduced time complexity, multiple figures, even crowds can be animated. In addition, calculations can be reused for the same actor and marker-set allowing different data sets to be blended. The main contributions in this dissertation are the new unbiased center formulae; the full statistical analysis of this new formula; and the analysis of when the best measurement conditions are to initiate the formula. The dissertation further establishes the application of these new formulae to motion capture to produce a real-time method of drawing skeletons of arbitrary articulated figures.

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