Chapter 1 INTRODUCTION

First of all, an explanation is needed for “articulated motion”. There are many examples of this kind of motion in everyday life. Just typing this paper involves articulated motion. Riding a bicycle involves articulated motion. These examples themselves are topics of research in their own right. An articulated figure can be formally defined as a collection of segments that are connected by joints. The type of joint defines its motion. If a joint rotates then the two segments can rotate around a point in up to three directions (i.e. degrees of freedom). If a joint can translate in space then the two segments can separate from each other in up to three directions adding more degrees of freedom. These motions for a joint, up to six degrees of freedom (DOF), are considered to be articulated motion – the topic at hand. The complications arise when more than one joint is in the figure. This creates multiple possibilities and there is no straightforward solution to the problem of analyzing the motion.

First, a computer representation of an articulated figure is needed. To keep physics in the equation, each segment of the figure must be solid. For physics related simulations, this is the only way to go since calculations of moments of inertia are fairly simple (cf. Chapter 7.2). A few other authors [97][113][121][122] use this approach for high-end physical simulations but most use triangulated surfaces for their “solid” figures. It is usually difficult to calculate moments of inertia from only the surface data. A tetrahedral solid mesh was chosen to simplify building up an arbitrary shape. With a solid shape, and its mass, the rotational physics and translational physics are fully calculable from solving the Newton-Euler differential equation. This unfortunately proves time-intensive.
More reliance on motion data can increase the speed. The motion data is usually made of raw position measurements of markers that have been placed on the surface of the body as in Figure 1.

![Markers on Actor](image)

*Figure 1 Markers on Actor*

Each marker in the data must be assigned (i.e. correlated) to a particular segment on the articulated figure. This can be done in one of two ways – autocorrelation and pre-
cognition. The latter is better for this research since autocorrelation is extremely time intensive. Precognition may sound like cheating but nine times out of ten, the user will know ahead of time which bits of data are associated with which segment on the articulated figure. A systematic approach must be thought of that allows the conclusive attachment of the segments to the data. This implies that the data must contain both relative position and orientation of the segment so the model can be sized and oriented into position. This satisfies an individual segment, but not the whole articulated figure. The rotation points in between the segments are needed. What this boils down to is that a stick figure needs to be drawn based solely on the motion-capture data. Whatever shape model can then be attached onto the stick figure. Existing methods were found in a few examples in the literature (cf. Chapter 8). The most popular was to perform a least-squares fit and a few still did non-linear fitting of the data. They all had one thing in common – they all assumed the marker moved on a sphere around the rotation point of the joint. This means that, to draw a stick figure, the center of a sphere must be solved at every joint. A little research shows there were only three major algorithms that solve the center of a sphere from data points sitting on the sphere. The most popular was the Maximum-Likelihood Estimator (MLE) that solved the problem in a non-linear fashion. This had the undesirable effect of being excruciatingly slow and sometimes not producing an answer at all. The next most popular type was linear least-squares solutions that involve fairly slow (but faster than MLE) pseudo-inverse matrix solutions. This was the preferred method by most authors concerned with speed. The third type of solution was a small set of approximations that were closed-form solutions to the best-fit sphere. These
formulae were unpopular because of biases and usually used only in the case to start off a non-linear search for the real answer.

1.1 Articulated Figure Animation

An articulated figure has a simple definition. It is defined as a set of jointed segments. The joint in between two segments can be free to rotate in one, two, or three directions and translate in one, two, or three directions. These types of movements are a rating system called the degrees of freedom (DOF). One DOF means the joint is free to rotate around only one axis. Two DOF and three DOF are similar. Four DOF joints are free to rotate around all three orthogonal axes and translate along one of them. Five and six add more translational freedom. A full six DOF joint has no restrictions in movement.

An articulated figure is made of a set of linked segments. A human figure and many other animal forms have a tree-like set of connected segments. Realistically, each segment is not some independent object attached at a specific point in space. Examining the human shoulder (see Figure 2) will uncover a complex joint system involving the scapula (shoulder blade), clavicle, and the humerus (upper arm bone).
Not only does each of these bones rotate around sockets but also have small amounts of translational freedom. Even with this complexity, certain levels of approximation can make this joint look like it is capable of simple three DOF motion. It is also customary to approximate the elbow joint (see Figure 3) with a one DOF model even though there are three bones involved (humerus, radius, ulna).
Full physics based animation must present a suitable musculoskeletal model that produces the level of approximation desired for the figure. The attachments of muscles to the skeleton must be well defined. Forward kinetics is the study of motion by applying forces on a skeleton to produce the joint motion. This method of animation is usually used in simulators. Inverse kinetics is backwards to this approach and calculates forces that are necessary to make a joint move from one position to another. The ability for this method to get from one pose to the next is what makes it ideal for cartoon or game animation when key framing is used. Kinematics is different from kinetics in that it uses angular motion instead of forces. Forward kinematics produces skeletal motion by apply-
ing joint rotations from a specific motion model. Inverse kinematics produces the desired pose by calculating the necessary angular movements.

1.2 Motion Capture Systems

Studying articulated motion is not possible without capturing positions of real motion during the activities. There have been many methods devised to capture positional information. The most common information that is captured is the three-dimensional position of specific points on the articulated figure. There are various commercial systems for 3D positions that involve many technologies. zFlo, Inc. (http://www.zflomotion.com/) has a video based system. The Basler A 600 series of digital cameras can take 105 frames per second.

Three or more of these cameras are placed around the actor in such a way to be able to see the markers on the body throughout the motion under study. After the videos are taken, the markers can be triangulated assuming that the camera positions are previously measured or calibrated. zFlo provides software for this analysis (Figure 4).
Another more popular video capture system is BodyBuilder (Figure 5) from Vicon Peak (http://www.vicon.com). Their system has been around for more than ten years and has been used in movies by Industrial Light and Magic. Their software is considered robust with many man-years of software development.
1.3 Symbols and Conventions

The mathematical symbols and acronyms that are used consistently through this paper are presented in the following table. The definitions are presented later when they are first used.

- $O(y)$ .................................................. on the order of (i.e. size of) $y$
- $N$ .................................................. number of measurements
- $CRLB$ .................................................. Cramér-Rao Lower Bound covariance of estimator
- $D$ .................................................. dimension of hypersphere
- $FLOP$ .................................................. floating point operation
- $x_i$ .................................................. $i^{th}$ measurement of position
- $\bar{x}$ .................................................. average of all measurements of positions

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\( \mu_i \) ..................expectation of the \( i^{th} \) position
\( \bar{\mu} \) ..................average of expectations of all positions
\( \Sigma \) ..................measurement covariance of each position
\( \hat{\Sigma} \) ..................measurement covariance estimator
\( \sigma \) ..................measurement standard deviation
\( C \) ..................sample covariance
\( C_0 \) ..................true covariance of expected positions
\( S \) ..................sample third central vector moment
\( S_0 \) ..................true third central moment of expected positions
\( \hat{F}_0 \) ..................true fourth central moment of expected positions
\( \hat{c} \) ..................center estimator
\( c_0 \) ..................true center of hypersphere
\( \hat{r} \) ..................radius estimator
\( r_0 \) ..................true radius of hypersphere
\( \lambda \) ..................eigenvalue of matrix
\( v \) ..................eigenvector of matrix
\( \chi^T \) ..................transpose of column vector into row vector
\( A^{-1} \) ..................multiplicative inverse of matrix
\( A^T \) ..................transpose of matrix
\( \nabla \) ..................vector gradient operator
\( |x| \) ..................magnitude of vector
\( |A| \) ..................determinant of matrix
\( N_3(\mu, \Sigma) \) ..................three dimensional vector Normal Distribution
\( \rho(A) \) ..................spectral radius of matrix
\( Tr(A) \) ..................trace (sum of diagonals) of matrix
\( E(A) \) ..................expectation of random variate
\( Var(A) \) ..................variance of random variate
\( Cov(A,B) \) ..................covariance of two variates (matrix or scalar)