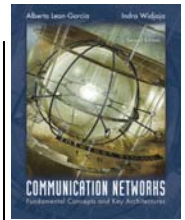
Chapter 3
Digital Transmission Fundamentals



Digital Representation of Information
Why Digital Communications?
Digital Representation of Analog Signals
Characterization of Communication Channels
Fundamental Limits in Digital Transmission
Line Coding
Modems and Digital Modulation
Properties of Media and Digital Transmission Systems
Error Detection and Correction

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Questions of Interest

- How long will it take to transmit a message?
  - How many bits are in the message (text, image)?
  - How fast does the network/system transfer information?
- Can a network/system handle a voice (video) call?
  - How many bits/second does voice/video require? At what quality?
- How long will it take to transmit a message without errors?
  - How are errors introduced?
  - How are errors detected and corrected?
- What transmission speed is possible over radio, copper cables, fiber, …?

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A Transmission System

Transmitter

- Converts information into *signal* suitable for transmission
- Injects energy into communications medium or channel
  - Telephone converts voice into electric current
  - Modem converts bits into tones

Receiver

- Receives energy from medium
- Converts received signal into form suitable for delivery to user
  - Telephone converts current into voice
  - Modem converts tones into bits

Transmission Impairments

Communication Channel

- Pair of copper wires
- Coaxial cable
- Radio
- Light in optical fiber
- Light in air
- Infrared

Transmission Impairments

- Signal attenuation
- Signal distortion
- Spurious noise
- Interference from other signals

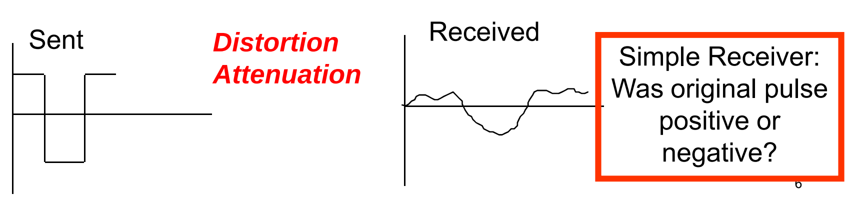
Analog Long-Distance Communications

- Each repeater attempts to restore analog signal to its original form
- Restoration is imperfect
  - Distortion is not completely eliminated
  - Noise & interference is only partially removed
- Signal quality decreases with # of repeaters
- Communications is distance-limited
- Still used in analog cable TV systems
- Analogy: Copy a song using a cassette recorder

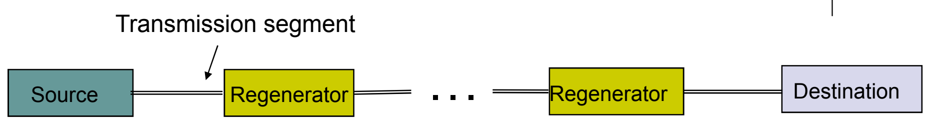
Analog vs. Digital Transmission

Analog transmission: all details must be reproduced accurately

Digital transmission: only discrete levels need to be reproduced



Digital Long-Distance Communications



- Regenerator recovers original data sequence and retransmits on next segment
- Can design so error probability is very small
- Then each regeneration is like the first time!
- Analogy: copy an MP3 file
- Communications is possible over very long distances
- Digital systems vs. analog systems
  - Less power, longer distances, lower system cost
  - Monitoring, multiplexing, coding, encryption, protocols…

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Bit Rates of Digital Transmission Systems

System	Bit Rate	Observations
Telephone twisted pair	33.6-56 kbps	4 kHz telephone channel
Ethernet twisted pair	10 Mbps, 100 Mbps	100 meters of unshielded twisted copper wire pair
Cable modem	500 kbps-4 Mbps	Shared CATV return channel
ADSL twisted pair	64-640 kbps in, 1.536-6.144 Mbps out	Coexists with analog telephone signal
2.4 GHz radio	2-11 Mbps	IEEE 802.11 wireless LAN
28 GHz radio	1.5-45 Mbps	5 km multipoint radio
Optical fiber	2.5-10 Gbps	1 wavelength
Optical fiber	>1600 Gbps	Many wavelengths

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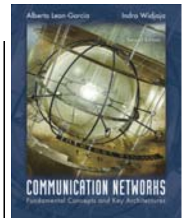

Properties of Media and Digital Transmission Systems

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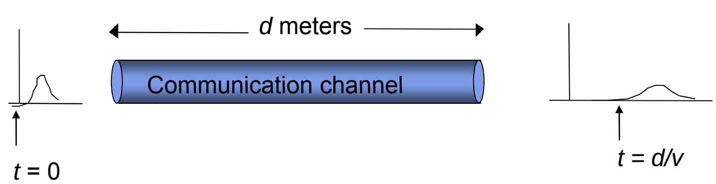
Fundamental Issues



- Propagation speed of signal
  - c = 3 x 10^8 meters/second in vacuum
  - $\nu = c/\sqrt{\varepsilon}$ speed of light in medium where $\varepsilon > 1$ is the dielectric constant of the medium
  - ν = 2.3 x 10^8 m/sec in copper wire; ν = 2.0 x 10^8 m/sec in optical fiber

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Twisted Pair

A twisted pair consists of two insulated copper wires, typically about 1mm thick

- More twists per cm leads to less crosstalk and better quality over longer distance

(a)

(b)

(a) Category 3 UTP (16 MHz).   (b) Category 5 UTP (100 MHz).

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Twisted Pair Bit Rates

Table 3.5 Data rates of 24-gauge twisted pair

Standard	Data Rate	Distance
T-1	1.544 Mbps	18,000 feet, 5.5 km
DS2	6.312 Mbps	12,000 feet, 3.7 km
1/4 STS-1	12.960 Mbps	4500 feet, 1.4 km
1/2 STS-1	25.920 Mbps	3000 feet, 0.9 km
STS-1	51.840 Mbps	1000 feet, 300 m

- Twisted pairs can provide high bit rates at short distances
- Asymmetric Digital Subscriber Loop (ADSL)
  - High-speed Internet Access
  - Lower 3 kHz for voice
  - Upper band for data
  - 64 kbps inbound
  - 640 kbps outbound
- Much higher rates possible at shorter distances
  - Strategy for telephone companies is to bring fiber close to home & then twisted pair
  - Higher-speed access + video

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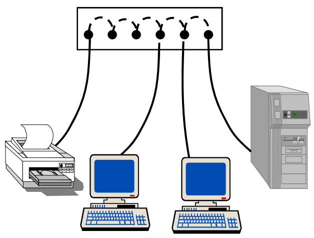
Ethernet LANs



- Category 3 unshielded twisted pair (UTP): ordinary telephone wires
- Category 5 UTP: tighter twisting to improve signal quality
- Shielded twisted pair (STP): to minimize interference; costly
- 10BASE-T Ethernet
  - 10 Mbps, Baseband, Twisted pair
  - Two Cat3 pairs
  - Manchester coding, 100 meters
- 100BASE-T4 *Fast* Ethernet
  - 100 Mbps, Baseband, Twisted pair
  - Four Cat3 pairs
  - Three pairs for one direction at-a-time
  - 100/3 Mbps per pair;
  - 3B6T line code, 100 meters
- Cat5 & STP provide other options

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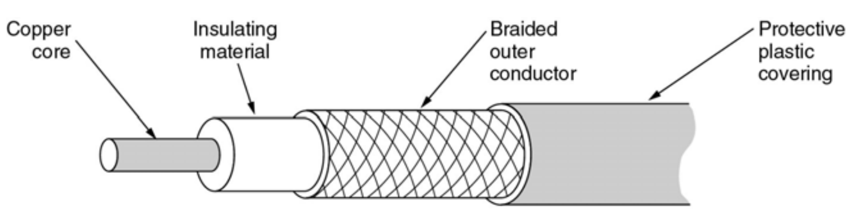
Coaxial Cable

- A good combination of high bandwidth and excellent interference immunity
  - Higher bandwidth than twisted pair
  - Cable TV distribution
  - Long distance telephone transmission
  - Original Ethernet LAN medium



Coaxial Cable (Cont.)

- A constant bit rate video stream requires transmitting 30 screen images (frames) per second. The screen is 480 X 640 pixels, each pixel being 24 bits. How much bandwidth is needed for a coaxial cable?

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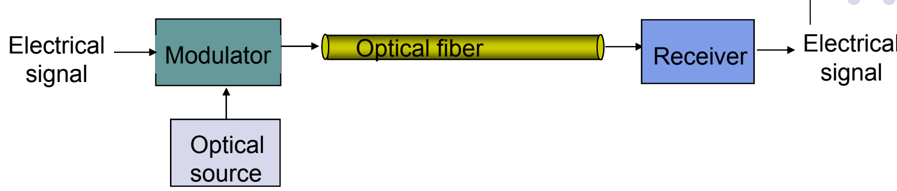
Optical Fiber



- Light sources (lasers, LEDs) generate pulses of light that are transmitted on optical fiber
  - Very long distances (>1000 km)
  - Very high speeds (>40 Gbps/wavelength)
  - Nearly error-free (BER of 10^{-15})
- Profound influence on network architecture
  - Dominates long distance transmission
  - Distance less of a cost factor in communications
  - Plentiful bandwidth for new services

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Optical Fiber Properties

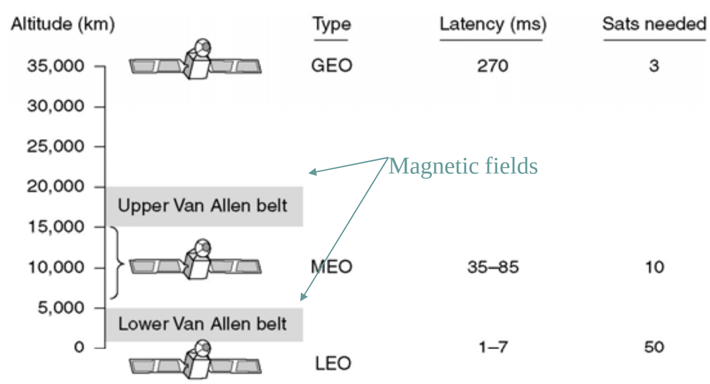
Advantages

- ***Very low attenuation***
- ***Noise immunity***
- ***Extremely high bandwidth***
- Security: very difficult to tap without breaking
- No corrosion
- More compact & lighter than copper wire

Disadvantages

- New types of optical signal impairments & dispersion
  - Wavelength dependence
- Limited bend radius
  - If physical arc of cable too high, light lost or won't reflect
  - Will break
- Difficult to splice
- Mechanical vibration becomes signal noise

Communication Satellites



Communication satellites and some of their properties, including altitude above the earth, round-trip delay time and number of satellites needed for global coverage.

Where are the 24 GPS satellites?

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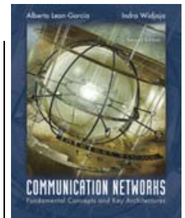

Error Detection and Correction

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Error Control

- Digital transmission systems introduce errors
- Applications require certain reliability level
  - Data applications require error-free transfer
  - Voice & video applications tolerate some errors
- Error control used when transmission system does *not* meet application requirement
- Error control ensures a data stream is transmitted to a certain level of accuracy despite errors
- Two basic approaches:
  - Error ***detection*** & retransmission
  - Error ***correction***

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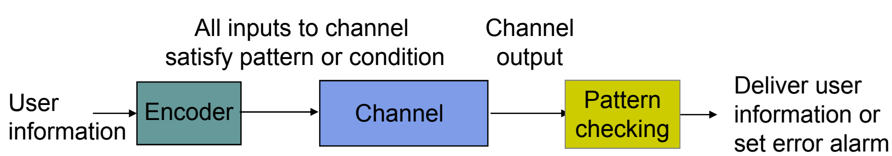
Codeword and Hamming Distance

- A n-bit codeword: a frame of m-bit data plus k-bit redundant check bits (n = m + k)
- The number of bit positions in which two codewords differ is called the Hamming distance.
  - Example: 10001001 and 10110001

Key Idea

- All transmitted data blocks ("codewords") satisfy a pattern
  - If received block doesn't satisfy pattern, it is in error
  - Redundancy(r)
- Blindspot: when channel transforms a codeword into another codeword



Error Detecting Codes – Single Parity bit

- Parity bit: to make the number of 1 bits in a codeword even or odd ($k = 1$)
  - Examples

Can a parity bit used to detect a single-bit error in a codeword?

Can a parity bit used to detect a double-bit error in a codeword? Triple…?

What is the hamming distance of the two codewords (before and after error)?

Can a parity bit used to correct a single-bit error in a codeword?

Parity bit used in ASCII code

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How good is the single parity check code?

- *Redundancy*: Single parity check code adds 1 redundant bit per m information bits: overhead = $1/(m + 1)$
- *Coverage*: all error patterns with odd # of errors can be detected
  - An error patten is a binary $(m + 1)$-tuple with 1s where errors occur and 0's elsewhere
  - Of 2^{k+1} binary $(m + 1)$-tuples, ½ are odd, so 50% of error patterns can be detected
- Is it possible to detect more errors if we add more check bits?
- Yes, with the right codes

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What if bit errors are random?

- Many transmission channels introduce bit errors at random, independently of each other, and with probability p
- Some error patterns are more probable than others:

$$P[10000000] = p(1-p)^7 = (1-p)^8 \left(\frac{p}{1-p}\right) \text{ and}$$

$$P[11000000] = p^2(1-p)^6 = (1-p)^8 \left(\frac{p}{1-p}\right)^2$$

- In any worthwhile channel $p < 0.5$, and so $(p/(1-p) < 1)$
- It follows that patterns with 1 error are more likely than patterns with 2 errors and so forth
- What is the probability that an undetectable error pattern occurs?

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Single parity check code with random bit errors

- Undetectable error pattern if even # of bit errors:

$$P[\text{error detection failure}] = P[\text{undetectable error pattern}]$$
$$= P[\text{error patterns with even number of 1s}]$$

$$= \binom{n}{2} p^2(1-p)^{n-2} + \binom{n}{4} p^4(1-p)^{n-4} + \ldots$$

- Example: Evaluate above for $n = 32$, $p = 10^{-3}$

$$P[\text{undetectable error}] = \binom{32}{2} (10^{-3})^2 (1-10^{-3})^{30} + \binom{32}{4} (10^{-3})^4 (1-10^{-3})^{28}$$

$$\approx 496\ (10^{-6}) + 35960\ (10^{-12}) \approx 4.96\ (10^{-4})$$

- For this example, roughly 1 in 2000 error patterns is undetectable

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Two-Dimensional Parity Check

- More parity bits to improve coverage
- Arrange information as columns
- Add single parity bit to each column
- Add a final "parity" column
- Used in early error control systems

1	0	0	1	0	0
0	1	0	0	0	1
1	0	0	1	0	0
1	1	0	1	1	0
1	0	0	1	1	1

Last column consists of check bits for each row

Bottom row consists of check bit for each column

What is its error-detecting capability? How about its error-correction capability

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Other Error Detection Codes

- Many applications require very low error rate
- Need codes that detect the vast majority of errors
- Single parity check codes do not detect enough errors
- Two-dimensional codes require too many check bits
- The following error detecting codes used in practice:
  - Internet Check Sums
  - CRC Polynomial Codes

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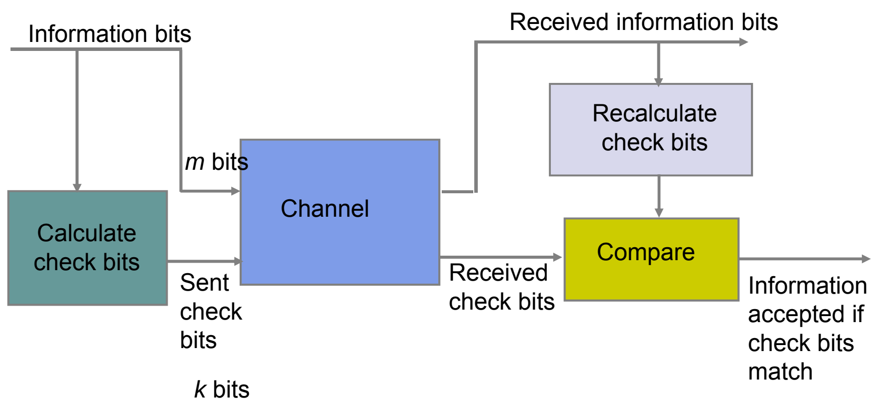
Checkbits & Error Detection



What are good patterns to use?

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Internet (IP) Checksum

- Let *IP header* consist of L, 16-bit words, $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{L-1}$
- The algorithm appends a 16-bit checksum $\mathbf{b}_L$

The checksum $\mathbf{b}_L$ is calculated as follows:

- Treating each 16-bit word as an integer, find
  $$\mathbf{x} = (\mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + \ldots + \mathbf{b}_{L-1}) \text{ modulo } 2^{16}-1$$
- The checksum is then given by:
  $$\mathbf{b}_L = -\mathbf{x}$$

Thus, the headers must satisfy the following ***pattern***:

$$\mathbf{0} = (\mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + \ldots + \mathbf{b}_{L-1} + \mathbf{b}_L) \text{ modulo } 2^{16}-1$$

- The checksum calculation is carried out in software using one's complement arithmetic

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Internet Checksum Example

Assume 4-bit words
Use mod 2^4-1 arithmetic
$\underline{b}_0$=1100 = 12
$\underline{b}_1$=1010 = 10

Use Modulo Arithmetic

Use Binary Arithmetic

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Internet Checksum Example

Use Modulo Arithmetic

- Assume 4-bit words
- Use mod 2^4-1 arithmetic
- $\underline{b}_0$=1100 = 12
- $\underline{b}_1$=1010 = 10
- $\underline{b}_0+\underline{b}_1$=12+10=7 mod15
- $\underline{b}_2$ = -7 = 8 mod15
- Therefore
- $\underline{b}_2$=1000

Use Binary Arithmetic

- Note 16 mod15 =1
- So: 10000 mod15 = 0001
- leading bit wraps around

$b_0 + b_1$ = 1100+1010
=10110
=10000+0110
=0001+0110
=0111
=7

Take 1s complement

b_2 = -0111 =1000

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Polynomial Codes

- Polynomials instead of vectors for codewords
- Polynomial arithmetic instead of check sums
- Implemented using shift-register circuits
- Also called *cyclic redundancy check (CRC)* codes
- Most data communications standards use polynomial codes for error detection
- Polynomial codes also basis for powerful error-correction methods

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Binary Polynomial Arithmetic

- Binary vectors map to polynomials

$$(i_{k-1}, i_{k-2}, \ldots, i_2, i_1, i_0) \rightarrow i_{k-1}x^{k-1} + i_{k-2}x^{k-2} + \ldots + i_2x^2 + i_1x + i_0$$

Binary Addition:

$$(x^7 + x^6 + 1) + (x^6 + x^5)$$

Binary Multiplication:

$$(x + 1)(x^2 + x + 1)$$

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Binary Polynomial Arithmetic

- Binary vectors map to polynomials

$$(i_{k-1}, i_{k-2}, \ldots, i_2, i_1, i_0) \rightarrow i_{k-1}x^{k-1} + i_{k-2}x^{k-2} + \ldots + i_2x^2 + i_1x + i_0$$

Addition:

$$(x^7 + x^6 + 1) + (x^6 + x^5) = x^7 + x^6 + x^6 + x^5 + 1$$
$$= x^7 + (1+1)x^6 + x^5 + 1$$
$$= x^7 + x^5 + 1 \quad \text{since } 1+1 \bmod 2 = 0$$

Multiplication:

$$(x + 1)(x^2 + x + 1) = x(x^2 + x + 1) + 1(x^2 + x + 1)$$
$$= (x^3 + x^2 + x) + (x^2 + x + 1)$$
$$= x^3 + 1$$

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Error-Detecting Codes – CRC base

- *Cyclic Redundancy Check* (CRC) use polynomial code, which is based on treating bit strings as representation of polynomials with coefficients of 0 and 1 only.
- A k-bit frame is regarded as the coefficient list for a polynomial with k terms, ranging from x^k-1 to x^0. Such a polynomial is said to be of degree k-1
  - Example: 110001
  - What is its degree?
  - What are its polynomial and coefficients?
- Polynomial arithmetic is done by per-bit XOR

  Example: 10011011 + 11001010

  11110000 - 10100110

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CRC Idea

- Both the sender and the receiver agree upon a generator polynomial *G(x)* as 1 xxx…x 1 in advance.

  Given a frame of m bits (a polynomial *M(x)*), the idea of CRC is to append a checksum to the end pf the frame in such a way that the polynomial represented by the checksumed frame is divisible by *G(x)*. When the receiver gets the checksummed frame, it tries dividing it by *G(x)*. If there is a reminder, there has been a transmission error.

  What kind of errors can be detected?

  How the checksum is calculated?

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CRC Algorithm

- Let r be the degree of G(x). Append r 0s to the low-order end of the frame, resulting *x^r M(x)* .
- Divide the bit string of *G(x)* into the bit string of *x^r M(x)*, using modulo 2 division.
- Subtract the reminder from the bit string of *x^r M(x)* using modulo 2 subtraction. The result is the checksummed frame to be transmitted, called T(x).

T(x) must be divisible by G(x)!

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CRC Example

- Frame: 1101011011
- Generator: 10011

What is the generator polynomial?

What is the actual frame to be transmitted?

If the third/fourth bit from the left is inverted during transmission, how this error is detected (or not detected) at the receiver's end?

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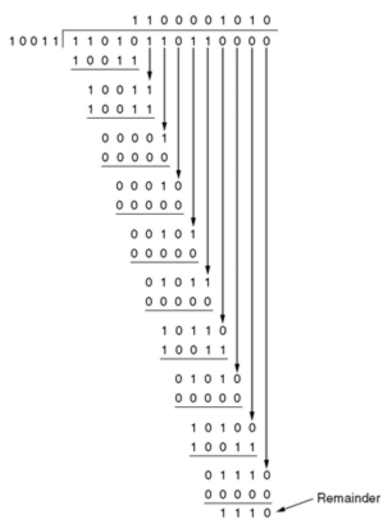
CRC Algorithm

Frame : 1 1 0 1 0 1 1 0 1 1
Generator: 1 0 0 1 1
Message after 4 zero bits are appended: 1 1 0 1 0 1 1 0 1 1 0 0 0 0



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Transmitted frame: 1 1 0 1 0 1 1 0 1 1 1 1 1 0

CRC Analysis

- What kind of errors will be detected?
- Imagine that a transmission error occurs, so that instead of *T(x)* arriving, *T(x) + E(x)* arrives. Each 1 bit in *E(x)* corresponds to a bit that has been inverted

  Example: 11001 (sent) ---- > 10101 (received)

  If *E(x)* is divisible by *G(x),* the error will slip by! So, how we select *G(x)*?!

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Designing good polynomial codes

- Select generator polynomial so that likely error patterns are not multiples of *g(x)*
- *Detecting Single Errors*
  - $e(x) = x^i$ for error in location $i + 1$
  - If *g(x)* has more than 1 term, it cannot divide x^i
- *Detecting Double Errors*
  - $e(x) = x^i + x^j = x^i(x^{j-i}+1)$ where $j>i$
  - If *g(x)* has more than 1 term, it cannot divide x^i
  - If g(x) is a *primitive* polynomial, it cannot divide x^m+1 for all $m<2^{n-k}-1$ (Need to keep codeword length less than $2^{n-k}-1$)
    - x^15+x^14+1 won't divide x^k + 1 for k < 32, 768
  - Primitive polynomials can be found by consulting coding theory books

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Designing good polynomial codes

- *Detecting Odd Numbers of Errors*
  - Suppose all codeword polynomials have an even # of 1s, then all odd numbers of errors can be detected
  - As well, $b(x)$ evaluated at $x = 1$ is zero because $b(x)$ has an even number of 1s
  - This implies $x + 1$ must be a factor of all $b(x)$
  - Pick $g(x) = (x + 1)\, p(x)$ where $p(x)$ is primitive

Standard Generator Polynomials

- CRC-8:

  $= x^8 + x^2 + x + 1$ ATM

- CRC-16:

  $= x^{16} + x^{15} + x^2 + 1$ Bisync

  $= (x + 1)(x^{15} + x + 1)$

- CCITT-16:

  $= x^{16} + x^{12} + x^5 + 1$ HDLC, XMODEM, V.41

- CCITT-32: IEEE 802, DoD, V.42

  $= x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$

Error Correcting Codes – An Error-correcting Code

- Given a complete list of the valid codewords, the minimum hamming distance of any two codewords is the hamming distance of the complete code
- Example: a complete code with four legal codewords of 0000000000, 0000011111, 1111100000, 1111111111
  - What is the hamming distance of the code?
  - How many error-bits at most can it correct?
  - How many error-bits at most can it detect?
  - What is the hamming distance of a code with a parity bit?
  - What is the relationship between the hamming distance and the number of error-bits to be detected and corrected?

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Error Correcting Codes – Low Limit on k

- A n-bit codeword: a frame of m-bit data plus k-bit redundant check bits (n = m + k)
- What is the lower limit on the number of bits k needed to correct single-bit errors in a n -bit codeword?

  (n+1) 2^m <= 2^n

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Hamming Method

- A n-bit codeword: a frame of m-bit data plus k-bit redundant check bits (n = m + k)
- Use of a Hamming code to detect & correct a single-bit error in a codeword
  - The bits that are powers of 2 are used as check bits.
  - The rest are filled up with the data bits
  - Each check bit forces the parity of some collection of bits, including itself, to be even (or odd)
  - To see which check bits the data bit in position k contributes to, rewrite k as a sum of powers of 2
  - A bit is checks by just those check bits occurring in its expansion (11 = 1 + 2 + 8)

Hamming Example

- Example: a n-bit codeword containing a 7-bit data 1001000

1001000 → 00110010000 (even-parity used)

How to correct it if 00100010000 is received instead?

How to correct it if 00110010001 is received instead?

How many check bits needed to d&c a single error in a 10-bit message

Error-Correcting Codes – Burst Errors

- What to do if errors come in burst, instead of isolated single-bit errors?

Char.	ASCII	Check bits
H	1001000	00110010000
a	1100001	10111001001
m	1101101	11101010101
m	1101101	11101010101
i	1101001	01101011001
n	1101110	01101010110
g	1100111	01111001111
	0100000	10011000000
c	1100011	11111000011
o	1101111	10101011111
d	1100100	11111001100
e	1100101	00111000101

Order of bit transmission

What is the maximum length of a burst that can be corrected in a sequence of k codewords?

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Error Detecting Codes vs. Error Correcting Codes

- Consider a channel on which errors are isolated and the error rate is 10^-6. Let the block size (m) be 1000.

  How many total bits required to provide single-bit error corrections for 1 MB data?

  How many total bits required to provide the error detection + retransmission?

Why wireless networks prefer error correction while wired networks may go for error detection and retransmission?

What kind of applications prefer error correction instead of detection?

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